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Математика как моделирующая система: семиотический подход

MATHEMATICS AS A MODELING SYSTEM: A SEMIOTIC APPROACH

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Preface

Mathematics and semiotics share many intellectual features and interests, from the study of how representations stand for specific kinds of referents to philosophical considerations of how these interrelate with reality. Nonetheless, rarely have in-depth studies of this intrinsic relation between the two been undertaken, with a few notable exceptions (as will be discussed in this book). Especially relevant to the study of the nature of mathematics is the concept of *model* – a term and notion that is used widely in both disciplines. However, to the best of our knowledge the theory of models in semiotics, known as *Modeling Systems Theory*, has rarely, if ever, been applied to the study of mathematical modeling. The purpose of this book is to do exactly that, since it is our view that mathematics is a *de facto* modeling system in the semiotic sense and it is our hope that from this it will be possible to gain considerable insights into how mathematics works and achieves the discoveries and forms of knowledge that it has since the dawn of antiquity. Hopefully, this will allow both mathematicians and semioticians to pursue similar or analogous research objectives with regard to understanding the biological and cognitive etiology sign systems and their connection to reality.

People have always taken pleasure in numbers and used numerical ideas to carry out the practical counting and arithmetical routines of everyday life. One of the oldest mathematical texts, an Egyptian work called the *Rhind Papyrus* written around 1650 BCE, is a collection of mathematical problems composed (seemingly) for educational purposes and to bring out the practical value of numeration and geometrical thinking to everyday life. The papyrus is also, indirectly, a treatise in how to do mathematics with the aid of symbols and notational devices. In a fundamental way, it is a treatise on the semiotic nature of mathematics, showing how symbolism (notation, numeral systems, and so on) is the sum and substance of mathematical method (Danesi 2002). In our view, any approach to understanding the nature of mathematics should take this very fact into account. Of course, the author of the papyrus never used any notions that could be construed as semiotic. But the work is inherently semiotic in the way it introduces mathematical concepts and problems. As this shows, it could be argued that from the advent of mathematics as a distinct discipline, mathematicians have been doing semiotics without knowing it. The late Thomas Sebeok (1920–2001) would often point out that the list of those who did semiotics without knowing it would fill the pages of countless books. He referred to

this state of affairs as the "Monsieur Jourdain syndrome". Monsieur Jourdain was a character in Molière's *Bourgeois Gentilhomme* (1670) who, when told that he spoke good prose, replied that he was not aware that he was using prose. Analogously, one can say that since the dawn of history, mathematicians have been doing something of which they were not aware – semiotics. To combat the syndrome, the two authors of this book, together with other leading semioticians have founded a scholarly network at the Fields Institute for Research in Mathematical Sciences at the University of Toronto in 2012 to study the interface between semiotic theory and mathematics. This book is a product of an emerging semiotic mindset within mathematics itself.

In our first chapter we simply present an array of facts and historical anecdotes that show how semiotics, Modeling Systems Theory, and mathematical theory and practice are intrinsically intertwined, discussing some previous work in the semiotics of mathematics and delving into the main insights that can be gained by amalgamating semiotics with mathematical theorizing. The second chapter looks at the oppositional nature of basic mathematical ideas and models, discussing the general applications of structuralist opposition theory to mathematics. The third chapter deals with the pervasiveness and cognitive power of diagrammatic modeling in mathematics, utilizing Peirce's main notion of Existential Graphing as a backdrop to the discussion. The fourth chapter looks at the role of metaphor in mathematics, with a discussion of blending theory and metaform theory as frameworks for understanding the psychological processes involved in constructing many types of mathematical models. Finally, in the fifth chapter we will look at the relation between mathematical modeling and reality, that is, we will discuss whether or not mathematics is discovered or invented (or both). We conclude with a few general philosophical comments.

The reader might view many of the notions that we discuss in this book as "reformulations" of standard notions in mathematics and semiotics, pasting them together in our own particular way. We would like to suggest that this is so only in a coincidental way. Our claim is that by approaching mathematics from the standpoint of semiotics, it will put us in a better intellectual position to grasp and understand its basic structure, no matter how subjective the approach. We must also warn readers about what not to expect from this book. They will not find in it an in-depth treatment of mathematical or semiotic theory. Our goal is simply to show how the basic nature of mathematics can best be envisioned as a semiotic modeling activity. Finally, we should inform the reader that, since founding the Fields network, we have become enormously enthusiastic about the prospect of uniting semiotics and mathematics. We cannot but agree with Mark Twain, when he wrote that: "Intellectual work is misnamed; it is a pleasure, a dissipation, and is its own highest reward" (Twain 1889: 167).

Marcel Danesi and Mariana Bockarova, 2012